7.4 Noncoherent detection of bandpass binary signals

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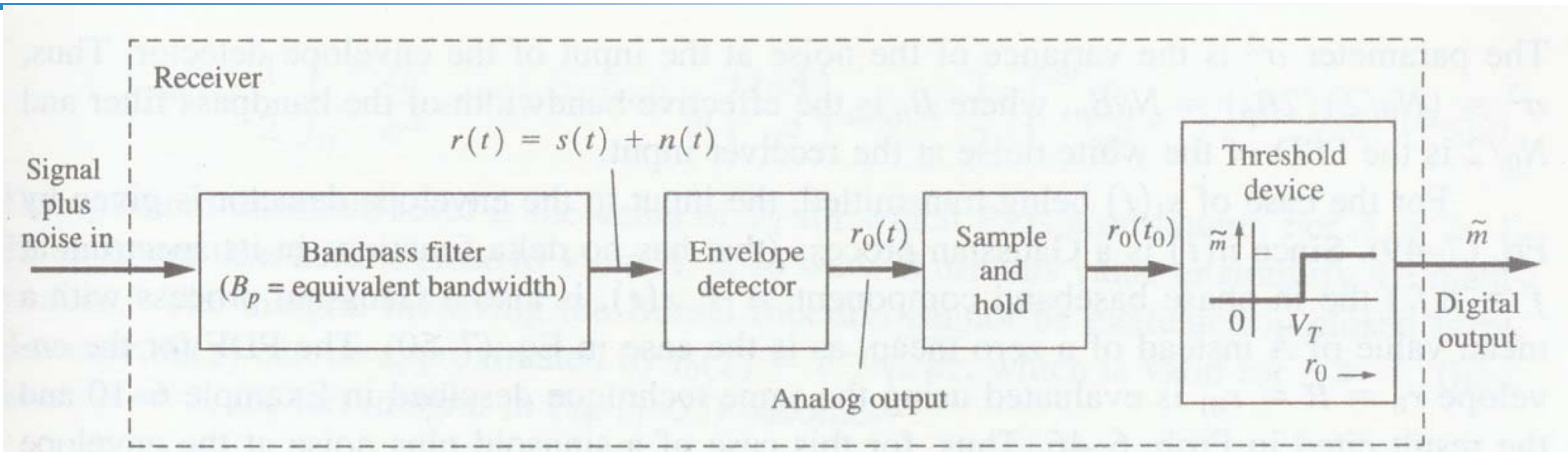

Figure 7–9 Noncoherent detection of OOK.

Let the bandwidth of the filter be B_p

The filter output: $r(t) = s(t) + n(t)$

$$n(t) = x(t)\cos(\omega_c t + \theta_c) - y(t)\sin(\omega_c t + \theta_c)$$

When:

$$s_1(t) = A\cos(\omega_c t + \theta_c), \qquad r_1(t) = [A + x(t)]\cos(\omega_c t + \theta_c) - y(t)\sin(\omega_c t + \theta_c)$$

$$s_2(t) = 0, \qquad r_2(t) = x(t)\cos(\omega_c t + \theta_c) - y(t)\sin(\omega_c t + \theta_c)$$

OOK

The BER is:

$$P_e = P(error \mid s_1 \ sent)P(s_1 \ sent) + P(error \mid s_2 \ sent)P(s_2 \ sent)$$
$$= \frac{1}{2}\int_{-\infty}^{V_T} f(r_0 \mid s_1)dr_0 + \frac{1}{2}\int_{V_T}^{\infty} f(r_0 \mid s_2)dr_0$$

The $r_2(t)$ envelope is a Rayleigh PDF:

$$f(r_0 \mid s_2) = \begin{cases} \dfrac{r_0}{\sigma^2} e^{-r_0^2/2\sigma^2}, & r_0 \geq 0 \\ 0, & r_0 < 0 \end{cases}$$

The $r_1(t)$ envelope is a Rician PDF:

$$f(r_0 \mid s_1) = \begin{cases} \dfrac{r_0}{\sigma^2} e^{-(r_0^2+A^2)/2\sigma^2} I_0\left(\dfrac{r_0 A}{\sigma^2}\right) & r_0 \geq 0 \\ 0 & r_0 < 0 \end{cases}$$

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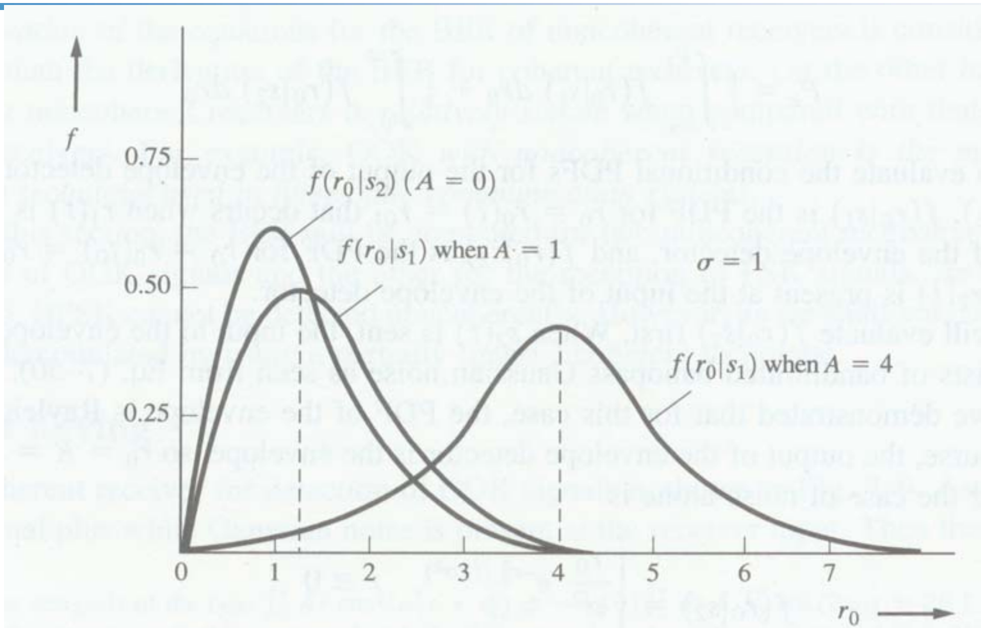


Figure 7–10 Conditional PDFs for noncoherent OOK reception.

for $A/\sigma >> 1 \quad \Leftrightarrow \quad \dfrac{E_b}{N_0} >> \dfrac{TB_p}{4}$

The BER for noncoherent detection of OOK is:

$$P_e = \frac{1}{2} e^{-\left[\left(\frac{1}{2TB_p}\right)\left(\frac{E_b}{N_0}\right)\right]}$$

FSK

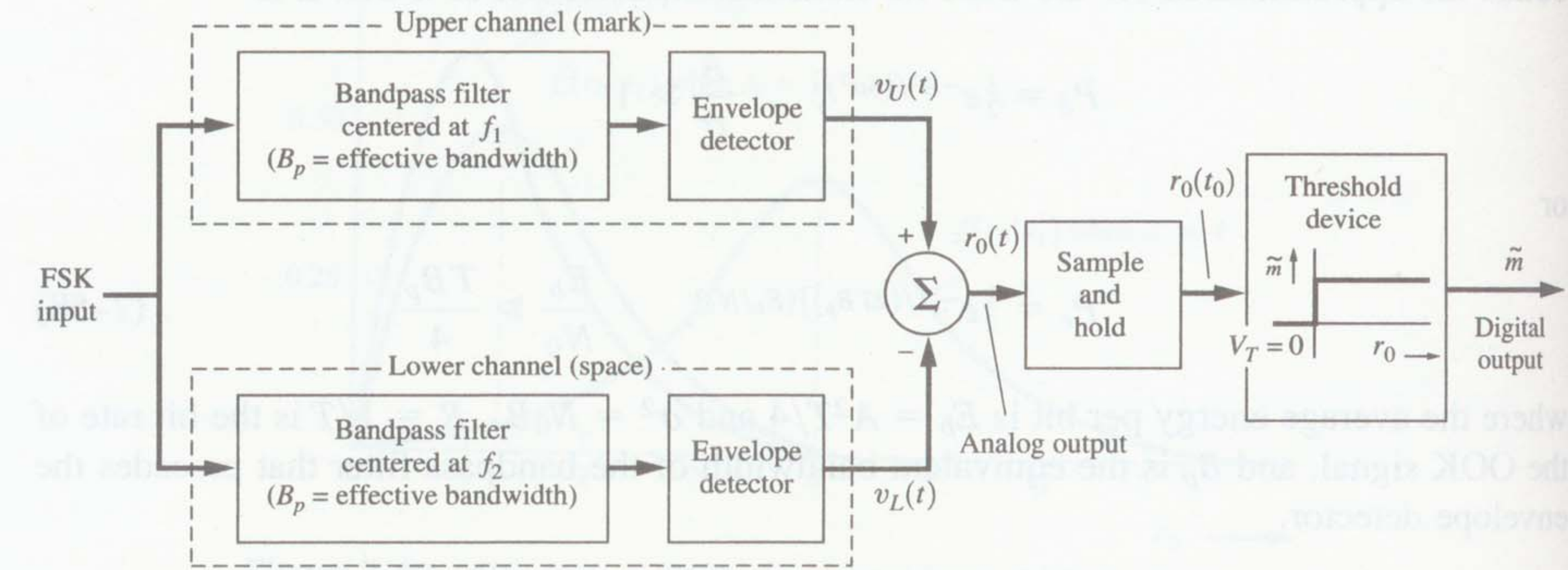


Figure 7–11 Noncoherent detection of FSK.

The input consists of an FSK signal plus white Gaussian noise

$$r(t) = \begin{cases} s_1(t) = A\cos(\omega_1 t + \theta_1), & data = 1 \\ s_2(t) = A\cos(\omega_2 t + \theta_2), & data = 0 \end{cases} + n(t)$$

FSK

Assume:

- **The bandpass filter bandwidth is B_p.**
- **The frequency shift $2\Delta F = f_1 - f_2$ is sufficiently large so that the spectra $s_1(t)$ and $s_2(t)$ have negligible overlap.**

The BER:

$$P_e = P(error \mid s_1\ sent)P(s_1\ sent) + P(error \mid s_2\ sent)P(s_2\ sent)$$

$$= \frac{1}{2}\int_{-\infty}^{V_T} f(r_0 \mid s_1)dr_0 + \frac{1}{2}\int_{V_T}^{\infty} f(r_0 \mid s_2)dr_0$$

$$= \int_{0}^{\infty} f(r_0 \mid s_2)dr_0$$

FSK

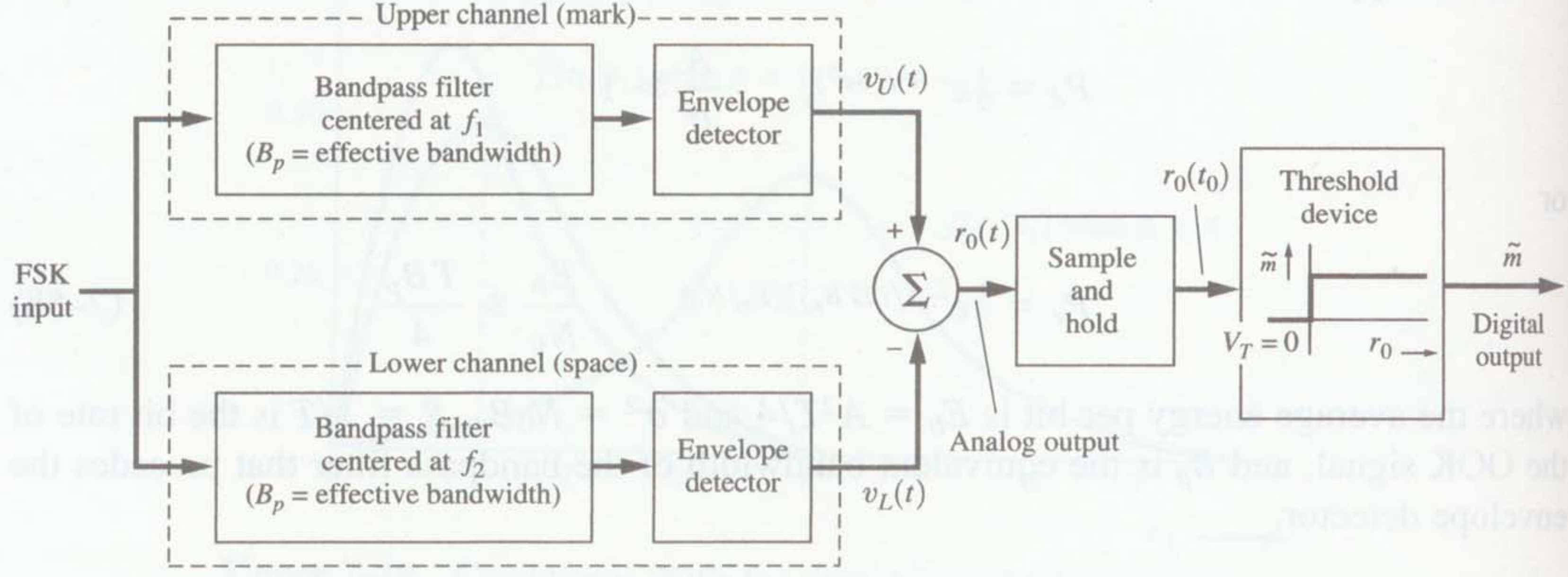


Figure 7–11 Noncoherent detection of FSK.

When $s_2(t)$ sent

- upper： The output is only Gaussian noise → Rayleigh *pdf*
- lower： The output is sinusoid plus noise → Rician *pdf*

The BER of FSK for noncoherent detection:

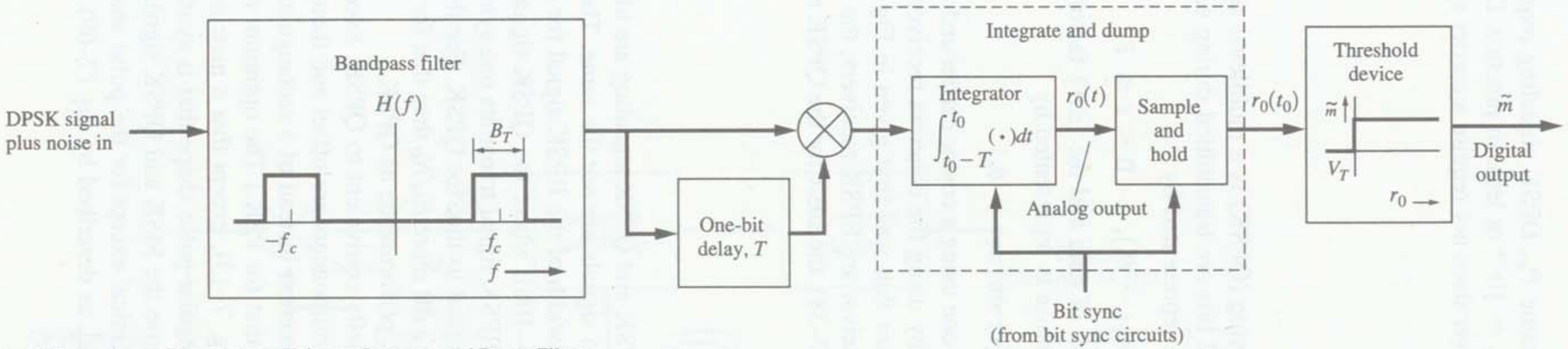
$$P_e = \frac{1}{2} e^{-A^2/(4\sigma^2)}$$

or

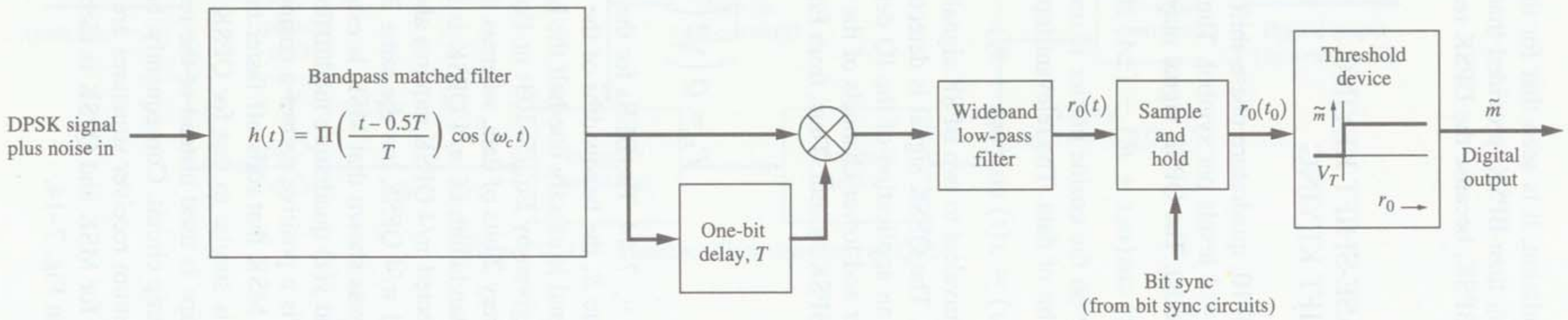
$$P_e = \frac{1}{2} e^{-\left[\left(\frac{1}{2TB_p}\right)\left(\frac{E_b}{N_0}\right)\right]}$$

DPSK

For PSK signals, a partially coherent technique can be used.



(a) A Suboptimum Demodulator Using an Integrate and Dump Filter



(b) An Optimum Demodulator Using a Bandpass Matched Filter

Figure 7–12 Demodulation of DPSK.

DPSK

Assume:

- **The additive input noise is white and Gaussian.**
- **The phase perturbation of the composite signal plus noise varies slowly so that the phase reference is essentially a constant.**
- **The transmitter carrier oscillator is sufficiently stable.**

For typical value of B_T and E_b/N_0 in the range of $B_T=3/T$ and $E_b/N_0=10$, the BER can be approximated by:

$$P_e = Q\left(\sqrt{E_b / N_0}\right)$$

The performance of the suboptimum receiver is similar to that obtaned for OOK and FSK.

DPSK

The BER for the optimum DPSK receiver of Fig. 7-12b is:

$$P_e = \frac{1}{2} e^{-(E_b/N_0)}$$

Comparison of the error performance between BPSK and DPSK with optimum demodulation:

For the same p_e (p_e=10^{-4} or less), DPSK signaling requires, at most, 1 dB more E_b/N_0 than BPSK.

7.5 Quadrature phase-shift keying and minimum-shift keying

QPSK

QPSK signal:

$$s(t) = (\pm A)\cos(\omega_c t + \theta_c) - (\pm A)\sin(\omega_c t + \theta_c)$$

The input noise is:

$$n(t) = x(t)\cos(\omega_c t + \theta_n) - y(t)\sin(\omega_c t + \theta_n)$$

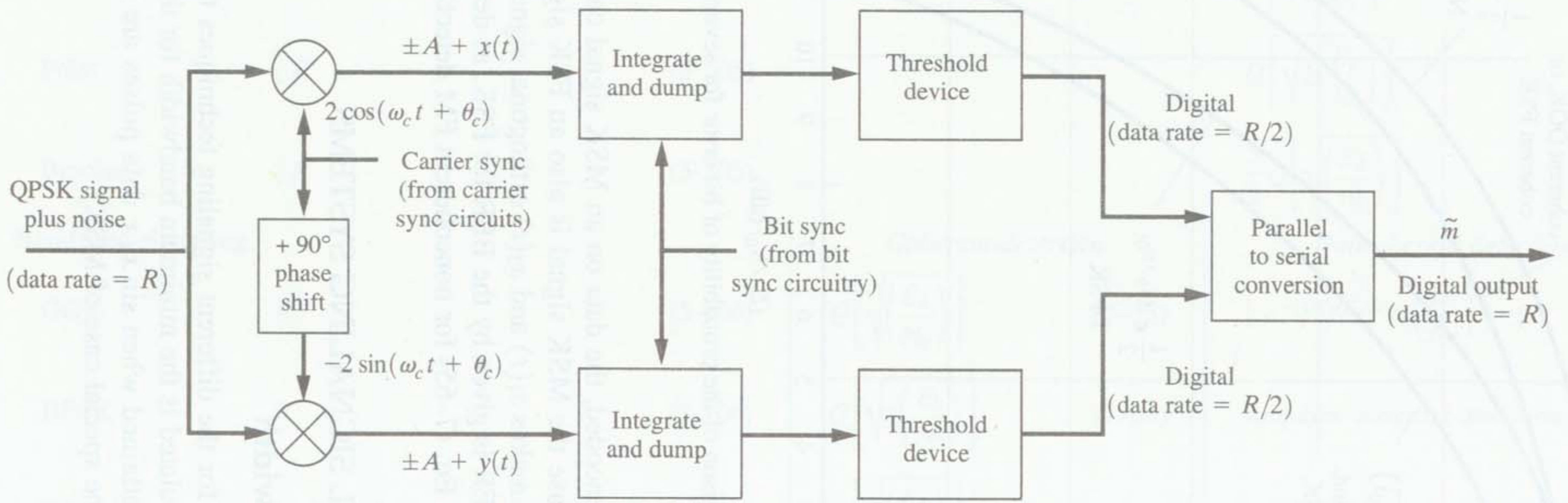


Figure 7–13 Matched-filter detection of QPSK.

QPSK

Because both the upper and lower channels of the receiver are BPSK receiver, the BER for the QPSK is:

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_0}\right)}\right)$$

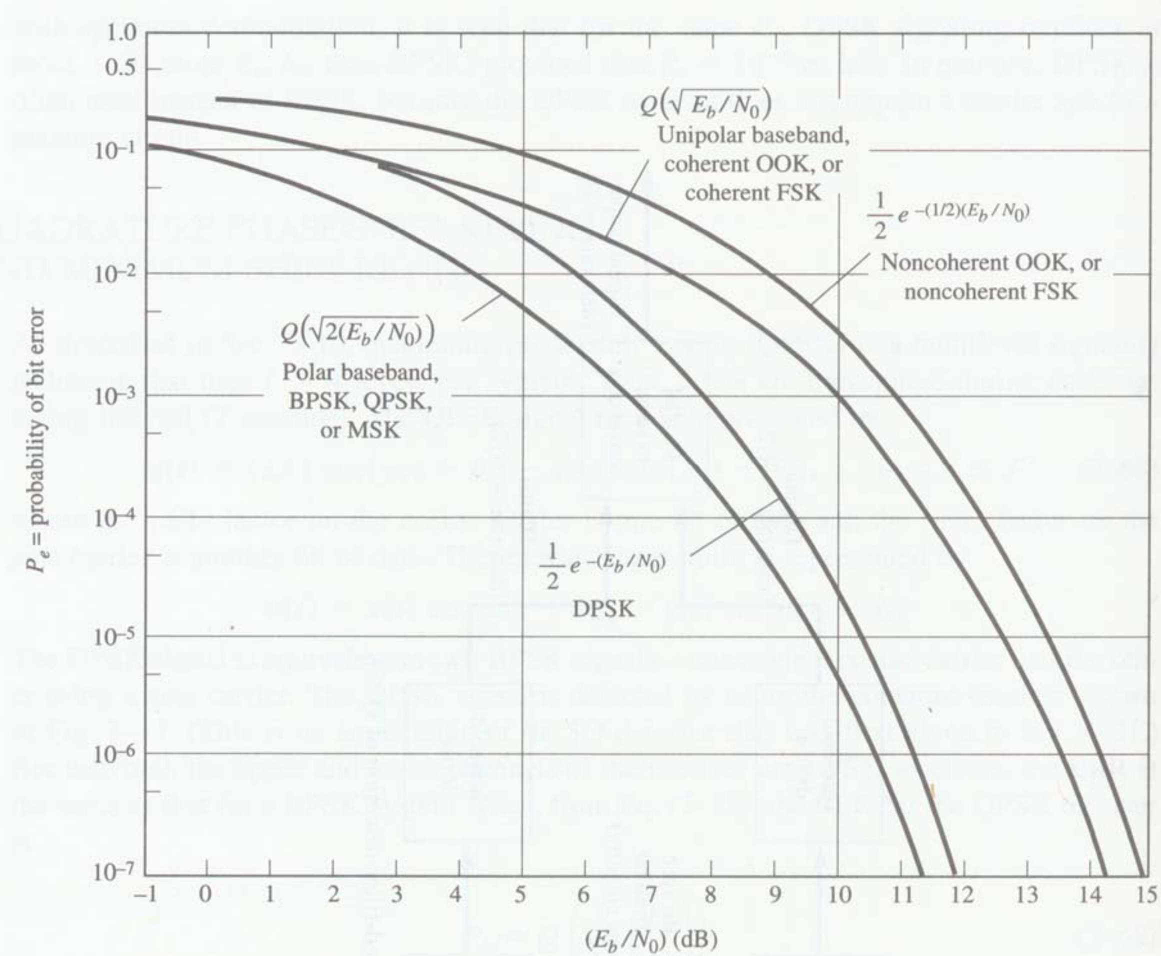


Figure 7–14 Comparison of the probability of bit error for several digital signaling schemes.

MSK

MSK is essentially equivalent to QPSK

Except

- The data on the $x(t)$ and $y(t)$ quadrature modulation components are **offset**.
- MSK equivalent **data pulse shape is a positive part of a cosine function** instead of a rectangular pulse.

Because the MSK and QPSK signal representations and the optimum receiver structures are identical except for the pulse shape, the BER for MSK and QPSK is identical:

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_0}\right)}\right)$$

MSK

If the data are properly encoded, the data on an MSK signal can also be detected by using FM-type detectors.

For this suboptimum detection of MSK, the BER is given by the BER for FSK:

For coherent FM detection:

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

For noncoherent FM detection:

$$P_e = \frac{1}{2} e^{-\left[\left(\frac{1}{2TB_p}\right)\left(\frac{E_b}{N_0}\right)\right]}$$

7.6 comparison of digital signaling systems

Comparison

Bit-error rate and bandwidth

TABLE 7–1 COMPARISON OF DIGITAL SIGNALING METHODS

Type of Digital Signaling	Minimum Transmission Bandwidth Required[a] (Where R Is the Bit Rate)		Error Performance			
Baseband signaling						
Unipolar	$\frac{1}{2}R$	(5-105)	$Q\left[\sqrt{\left(\frac{E_b}{N_0}\right)}\right]$			(7–24b)
Polar	$\frac{1}{2}R$	(5-105)	$Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)}\right]$			(7–26b)
Bipolar	$\frac{1}{2}R$	(5-105)	$\frac{3}{2}Q\left[\sqrt{\left(\frac{E_b}{N_0}\right)}\right]$			(7–28b)
Bandpass signaling			*Coherent detection*		*Noncoherent detection*	
OOK	R	(5-106)	$Q\left[\sqrt{\left(\frac{E_b}{N_0}\right)}\right]$	(7–33)	$\frac{1}{2}e^{-(1/2)(E_b/N_0)}, \left(\frac{E_b}{N_0}\right) > \frac{1}{4}$	(7–58)
BPSK	R	(5-106)	$Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)}\right]$	(7–38)	Requires coherent detection	
FSK	$2\Delta F + R$ where $2\Delta F = f_2 - f_1$ is the frequency shift	(5-89)	$Q\left[\sqrt{\left(\frac{E_b}{N_0}\right)}\right]$	(7–47)	$\frac{1}{2}e^{-(1/2)(E_b/N_0)}$	(7–65)
DPSK	R	(5-106)	Not used in practice		$\frac{1}{2}e^{-(E_b/N_0)}$	(7–67)
QPSK	$\frac{1}{2}R$	(5-106)	$Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)}\right]$	(7–69)	Requires coherent detection	
MSK	$1.5R$ (null bandwidth)	(5-115)	$Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)}\right]$	(7–69)	$\frac{1}{2}e^{-(1/2)(E_b/N_0)}$	(7–65)

[a] Typical bandwidth specifications by ITU are larger than these minima [Jordan, 1985].

Bit-error rate and bandwidth

Bit-error rate

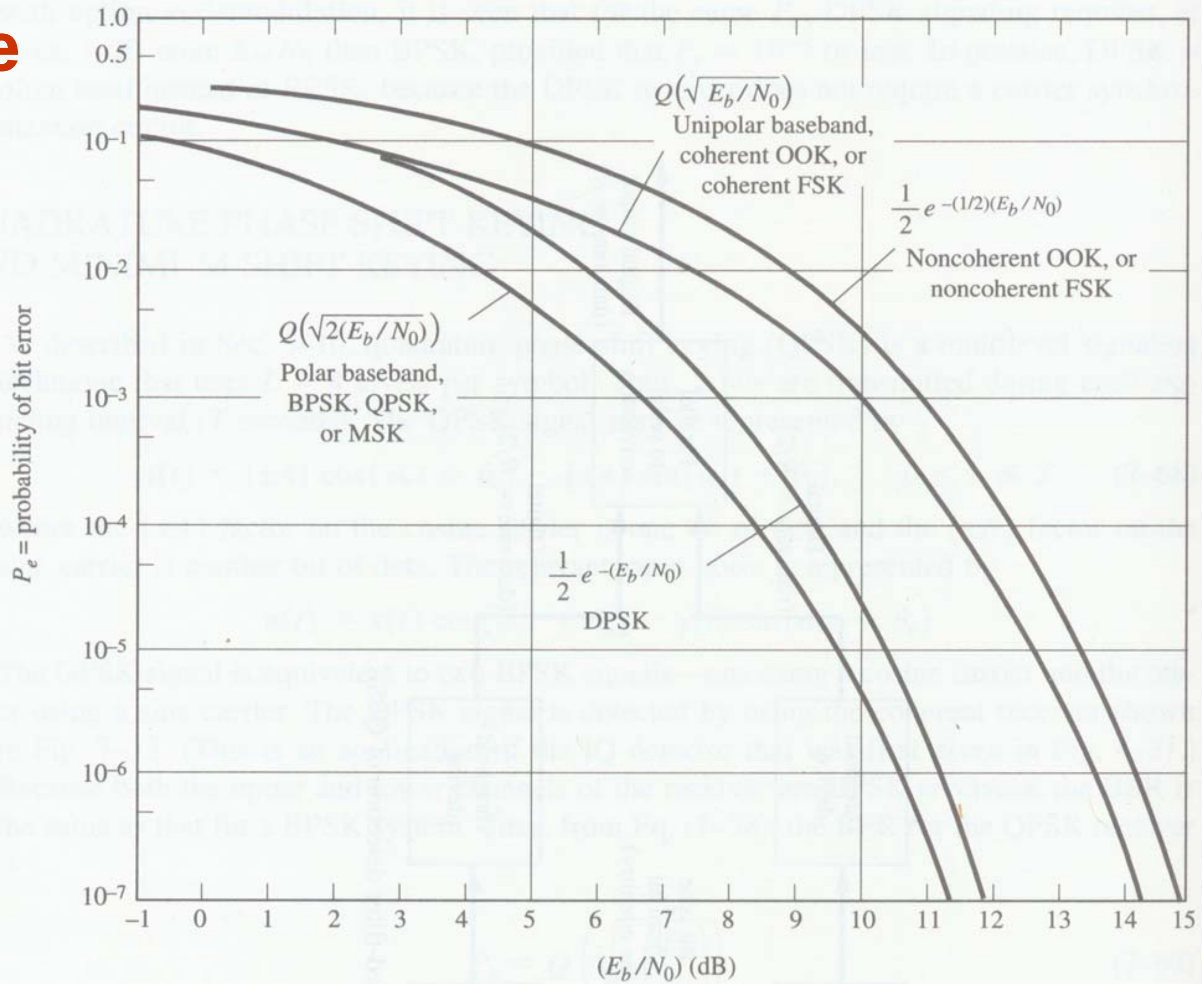


Figure 7–14 Comparison of the probability of bit error for several digital signaling schemes.

7.7 Output signal-to-noise ratio for PCM systems

Output SNR for PCM

Two main effects produce noise or distortion:

- **Quantizing noise** that is caused by the M-step quantized at PCM transmitter
- **Bit errors** in the recovered PCM signal . (channel noise, improper channel filtering, ISI etc.)

Output SNR for PCM

Question:

What is the peak signal power to average noise power ratio $(S/N)_{pk\ out}$ for the analog output?

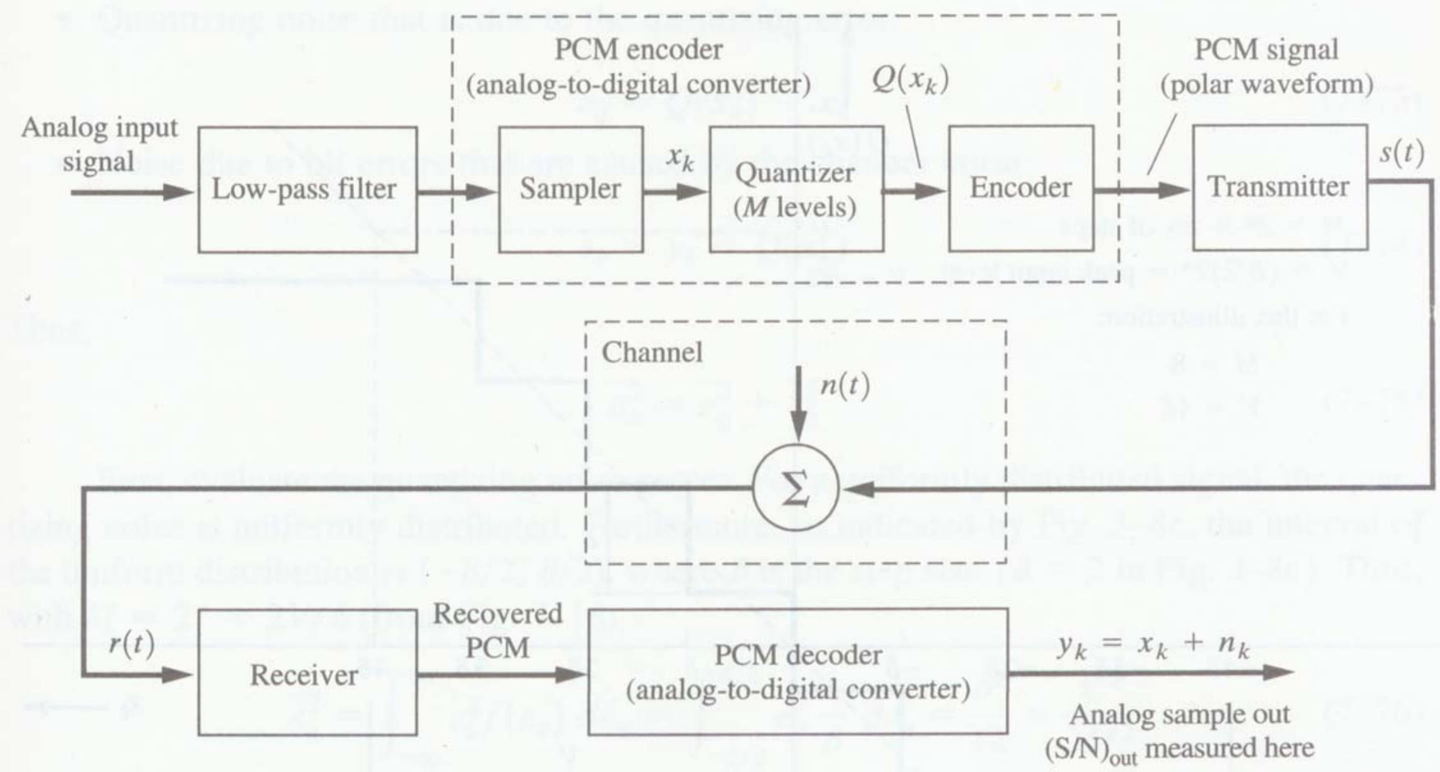
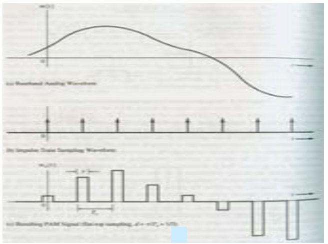
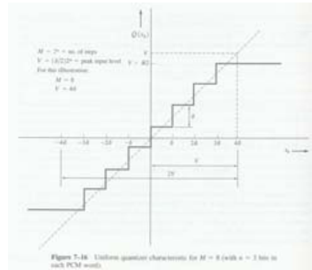


Figure 7–15 PCM communications system.

Output SNR for PCM





Sampling ($t=kt_s$)	Quantizing	Coding
x_k	$Q(x_k)$	$(a_{k1}, a_{k1}, \ldots a_{kn})$

…, 0001 0101, 1000 0101, …

- **Assume polar signaling is used and the PCM code words are related to the quantized values by**

$$Q(x_k) = V\sum_{j=1}^{n} a_{kj}\left(\frac{1}{2}\right)^{j}$$

Output SNR for PCM

The analog sample output of PCM system for the k-th sampling time is

$$y_k = x_k + n_k$$

The output peak signal power to average noise power ratio is

$$\left(\frac{S}{N}\right)_{pk\ out} = \frac{[(x_k)_{\max}]^2}{\overline{n_k^2}} = \frac{V^2}{\overline{n_k^2}}$$

where $\overline{n_k^2} = \overline{e_q^2} + \overline{e_b^2}$

Output SNR for PCM

- Quantizing noise that is due to the quantizing error

$$e_q = Q(x_k) - x_k$$

- Noise due to bit errors that are caused by the channel noise:

$$e_b = y_k - Q(x_k)$$

The quantizing noise power

$$\overline{e_q^2} = \int_{-\infty}^{+\infty} e_q^2 f(e_q) de_q = \int_{-\delta/2}^{+\delta/2} e_q^2 \frac{1}{\delta} de_q = \frac{\delta^2}{12} = \frac{V^2}{3M^2}$$

The noise power due to bit error

$$\overline{e_b^2} = \frac{4}{3} V^2 P_e \frac{M^2 - 1}{M^2}$$

Output SNR for PCM

Conclusion:

- The out **peak signal power to average noise power ratio** is

$$\left(\frac{S}{N}\right)_{pk\ out} = \frac{3M^2}{1+4(M^2-1)P_e}$$

- The **average-signal-to –noise ratio** is

$$\left(\frac{S}{N}\right)_{out} = \frac{\overline{(x_k)^2}}{\overline{n_k^2}} = \frac{V^2}{3\,\overline{n_k^2}} = \frac{1}{3}\left(\frac{S}{N}\right)_{pk\ out}$$

$$\left(\frac{S}{N}\right)_{out} = \frac{M^2}{1+4(M^2-1)P_e}$$

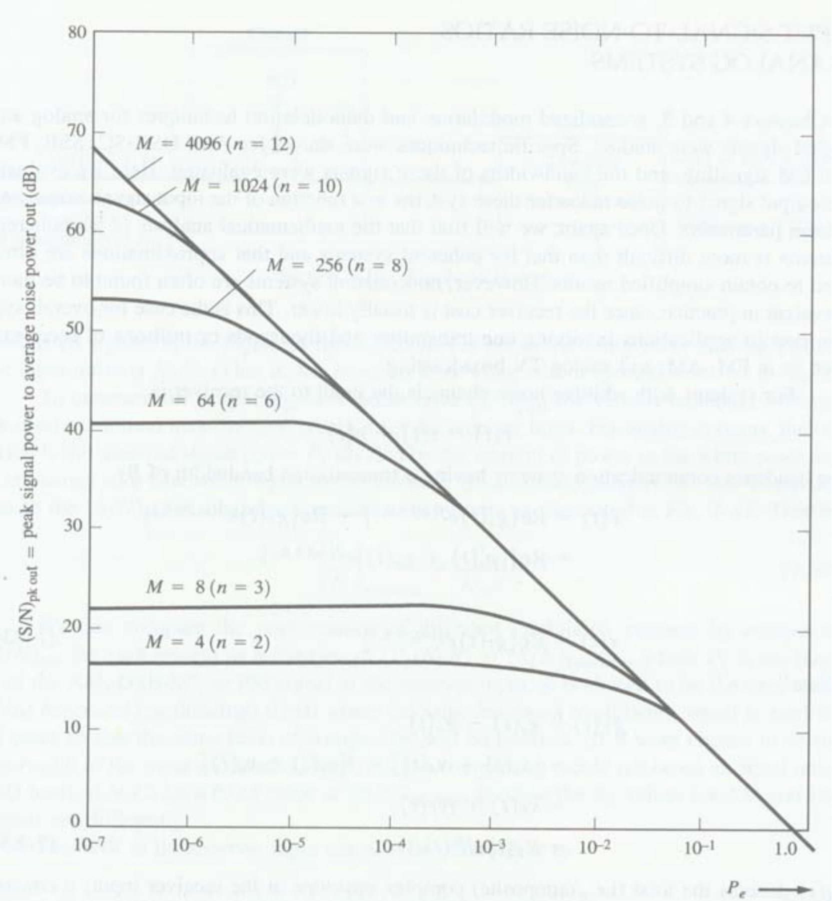


Figure 7–17 $(S/N)_{out}$ of a PCM system as a function of P_e and the number of quantizer steps M.